A new generalization of supersymmetric quantum mechanics to arbitrary dimensionality or number of distinguishable particles.

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Abstract

We present here a new approach to generalize supersymmetric quantum mechanics to treat multiparticle and multi-dimensional systems. We do this by introducing a *vector* superpotential in an orthogonal hyperspace. In the case of N distinguishable particles in three dimensions this results in a vector superpotential with $3N$ orthogonal components. The original scalar Schrödinger operator can be factored using a $3N$-component gradient operator and introducing vector "charge" operators: $\vec{Q}_1$ and $\vec{Q}_1^\dagger$. Using these operators, we can write the original (scalar) Hamiltonian as $H_1 = \vec{Q}_1^\dagger \cdot \vec{Q}_1 + E_0^{(1)}$ where E_0 is the ground state energy. The second sector Hamiltonian is a tensor given by $\overleftrightarrow{H}_2 = \vec{Q}_1 \vec{Q}_1^\dagger + E_0^{(1)}$ and is isospectral with H_1. The vector ground state of sector two, $\vec{\psi}_0^{(2)}$, can be used with the charge operator $\vec{Q}_1^\dagger$ to obtain the excited state wave functions of the first sector. In addition, we show that $\overleftrightarrow{H}_2$ can also be factored in

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terms of a sector two vector superpotential with components $W_{2j} = -\partial \ln \psi_{0j}^{(2)} / \partial x_j$. Here $\psi_{0j}^{(2)}$ is the j^{th} component of $\vec{\psi}_0^{(2)}$. Then one obtains charge operators $\vec{Q}_2$ and $\vec{Q}_2^\dagger$ so that the second sector Hamiltonian can be written as $\overleftrightarrow{H}_2 = \vec{Q}_2^\dagger \vec{Q}_2 + E_0^{(2)}$. This allows us to define a *third* sector Hamiltonian which is a *scalar*, $H_3 = \vec{Q}_2 \cdot \vec{Q}_2^\dagger + E_0^{(2)}$. This prescription continues with the sector Hamiltonians alternating between scalar and tensor forms accommodating both variational and Monte Carlo methods to obtain approximate solutions to both scalar and tensor sectors. We demonstrate the approach with examples of a pair of separable 1D harmonic oscillators and the example of a non-separable 2D anharmonic oscillator (or equivalently a pair of coupled 1D oscillators).